Single-telescope interferometry





Frantz Martinache, Laboratoire Lagrange, OCA

Marseille, 1873

Marseille, 1873



"Le grand télescope Foucault de l'Observatoire de Marseille"

Why would you want to mask what was then the best telescope available?

Le grand télescope Foucault, de l'Observatoire de Marseille.















 $V = (I_{\text{max}} - I_{\text{min}}) / (I_{\text{max}} + I_{\text{min}})$

 $\vee = |$





Fringe visibility





Marseille, 1873

Marseille, 1873





Edouard Stephan attempts the first stellar diameter measurement.

With a maximum baseline of 65 cm, he concludes about the diameter of stars

Ø★ < 0.158"









visibility:phase:0 < V < I $\Phi = \Phi_0$





pupil



pupil



pupil



pupil



pupil



pupil



pupil



interpretation is a bit harder... but in the Fourier space...











The mask geometry matters



The mask geometry matters



The mask geometry matters



same number of apertures... but more uv coverage (21 points instead of 9)

Redundancy






Redundancy













And a full aperture...



And a full aperture...

is very, very,

very, very,

very redundant



Atmosphere affects the phases Redundancy destroys the amplitudes















Toward an easier problem





Toward an easier problem



Toward an easier problem



To take advantage of self-calibration

9-hole mask
36 visibilities
84 triangles
(28 independent)





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9-hole mask
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84 triangles
(28 independent)





 $\Phi(1-2) = \Phi(1-2)_0 + (\Phi_1-\Phi_2)$ $\Phi(2-3) = \Phi(2-3)_0 + (\Phi_2-\Phi_3)$ $\Phi(3-1) = \Phi(3-1)_0 + (\Phi_3-\Phi_1)$

To take advantage of self-calibration

9-hole mask
36 visibilities
84 triangles
(28 independent)





 $\Phi(1-2) = \Phi(1-2)_0 + (\Phi_1 - \Phi_2)$ $\Phi(2-3) = \Phi(2-3)_0 + (\Phi_2 \cdot \Phi_3)$ $\Phi(3-1) = \Phi(3-1)_0 + (\Phi_3 - \Phi_1)$

An ideal observable: the closure-phase!

Jennison, 1958, MNRAS, 118, 276

Super-resolution with closure-phase



Martinache et al, 2009, ApJ, 695, 1183

Super-resolution with closure-phase





Martinache et al, 2009, ApJ, 695, 1183

Super-resolution with closure-phase





GJ 164 AB $M_1 = 0.247 +/- 0.019 M_s$ $M_2 = 0.096 +/- 0.008 M_s$

Martinache et al, 2009, ApJ, 695, 1183

Good for faint companions

40 % strehl 0.3 deg scatter stability ~ λ/1000 all passive !

Good for faint companions



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40 % strehl 0.3 deg scatter stability ~ λ/1000 all passive !

Understand your errors: statistical and systematic

Calibrate, calibrate, calibrate!



VAMPIRES

Norris et al, 2015, MNRAS, 447, 2894

Differential interfero-polarimetry



a) 1 tier - Wollaston prism only. No temporal variation leads to small error bars, but strong systematic errors (from non-common path) dominate.



b) 1 tier - LCVR only. No non-common path error, and the mean is ~ 1.0 . However since switching is slower than seeing temporal errors lead to large error bars.



c) 2 tiers - Wollaston + LCVR. The Wollaston and LCVR cancel each others errors. Systematic errors are still visible.



d) 3 tiers - Wollaston + LCVR + HWP. The HWP cancels out static systematic errors (such as those arising from instrumental effects). Here precision is limited by random error; additional integration time would improve precision further.

VAMPIRES

Norris et al, 2015, MNRAS, 447, 2894

The self-calibration properties of closure phase make NRM "bullet-proof"



NRM onboard JWST in the NIRISS instrument.

The self-calibration properties of closure phase make NRM "bullet-proof"



NRM onboard JWST in the NIRISS instrument.

Sivaramakrishnan et al, Astro2010T, 40

A game changer: AO







credit: Pr. James Lloyd

closure-phase planet direct detection LkCa 15 LkCa 15 disk শ্ব AU 50 AU (76 mas)

Transition-disk host star in Taurus association (150 pc) Companion detected @ 11 AU

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Kraus & Ireland, 2012, ApJ, 745, 5

If only one didn't have to mask...





interferogram image

uv-plane

If only one didn't have to mask...



Telescopes apertures are redundant

with AO, the phase can be linearized



with AO, the phase can be linearized



with AO, the phase can be linearized



the 3 basic configurations



I. the easy non-redundant one



$$\Phi = \Phi_0 + \mathbf{A} \phi$$

$$\Phi = \Phi_0 + \mathbf{A} \phi$$

$$\Phi^{AC} = \Phi^{AC}_0 + (\phi_{A} - \phi_{C})$$

$$\Phi^{BA} = \Phi^{BA}_0 + (\phi_{B} - \phi_{A})$$

$$\Phi = \begin{bmatrix} 0 & | & -| \\ | & 0 & -| \\ -| & | & 0 \end{bmatrix}$$

K = [I, I, I] verifies $K \cdot A = 0$: the closure phase
2. the first redundant case



 $\Phi^{AC} = \Phi^{AC}_{0} + (\varphi_{A} - \varphi_{C})$ $\Phi' = \operatorname{Arg}[\exp i(\Phi'_{0} + (\varphi_{A} - \varphi_{B})) + \exp i(\Phi'_{0} + (\varphi_{B} - \varphi_{C}))]$

2. the first redundant case



 $\Phi = \Phi_0 + \mathbf{R}^{-1} \mathbf{A} \boldsymbol{\varphi}$

$$\Phi^{AC} = \Phi^{AC}_{0} + (\phi_{A} - \phi_{C})$$

$$\Phi' = \Phi'_{0} + \frac{1}{2}(\phi_{A} - \phi_{C})$$

$$R^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 - 1 \\ 1 & 0 - 1 \end{bmatrix}$$

2. the first redundant case



$\Phi = \Phi_0 + \mathbf{R}^{-1} \mathbf{A} \boldsymbol{\varphi}$

$$\Phi^{AC} = \Phi^{AC}_{0} + (\phi_{A} - \phi_{C})$$

$$\Phi' = \Phi'_{0} + \frac{1}{2}(\phi_{A} - \phi_{C})$$

$$\Phi = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

K = [1, -2] verifies $K R^{-1}A = 0$: the kernel-phase

3. the real first case



$$\mathbf{R}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 1 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & -1 \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 1 & 0 & -1 & -2 \\ 1 & 0 & -1 & -2 \end{bmatrix}$$

You can extract self-calibrating observables from redundant apertures

This is of some consequence...



FIRST:

- 36-beam combiner
- fiber remapping
- spatial filtering

Perrin et al, 2006

This is of some consequence...



Redundant entrance pupil

Microlens array: fiber injection

Single-mode fibers: spatial filtering

Non redundant exit configuration

Prism: spectral dispersion

Focusing lens



Fringe patterns on the detector

FIRST:

- 36-beam combiner

Nith Agise problem is a false

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Perrin et al, 2006

For a complex array... use a computer



Kernel-phase

Find K so that $K \times A = 0$, but how?

Use the Singular Value Decomposition (SVD) of A:

$A = U \Sigma V^{\mathsf{T}}$

Rows of **K** form a basis for the left null space of **A** These new closure relations are called kernel-phases

Martinache, 2010, ApJ, 724, 464

I. Build a instrument model => A 2. Find the Kernel of A: K



Build a instrument model => A Find the Kernel of A: K

3. Fourier Transform each image 4. Extract phase φ



Build a instrument model => A Find the Kernel of A: K

3. Fourier Transform each image 4. Extract phase φ



5. Multiply K φ: you are done!

Additionally:

- statistics
- model the data (e.g. binary)
- determine contrast limits



Kernel-phase in space



Data @ I.9 μm (λ/D=I50 mas)

A ~10:1 contrast companion to a nearby Mdwarf identified with **milli-arc-second precision** at 0.5 λ /D

Martinache, 2010, ApJ, 724, 464





Original survey: Reid et al, 2006, 2008

Revisit ~ 80 brown dwarfs observed with HST/NIC1 in the F110W and F170M filters

- Doubled the fraction of known L-dwarf binary systems

- Improved astrometry x10

Grant HST-AR-12849.01-A

Pope et al, 2013, ApJ, 767, 110

Kernel-phase on ground based AO

Pope et al, 2015, in prep

Kernel-phase on ground based AO



- Separation: 131.4 +/- 0.9 mas
- Position Angle: 86.0 +/- 0.2 deg
- Contrast: 19.7 +/- 0.4

Data, courtesy of S. Hinkley Pope et al, 2015, in prep



Hinkley et al, 2011, ApJ, 726, 104

Better than the kernel-phase...



... are the statistically independent kernel-phases!

θ = **S.K.**φ

Ireland, 2013, MNRAS, 433, 1718

Requires empirical covariance matrices Part of a new file exchange standard?

Redundant is good for high contrast



Projected probability density function 120 140 80 100 Angular sep (mas) Projected probability density function 80 100 120 140 Angular sep (mas) Projected probability density function 350 250 300 Position Angle (deg) Redundant

Simulations for Palomar Hale Telescope PHARO full H-band filter

Strehl: 80 % Δ mag ~ 7 @ I.5 λ /D Calibrated closure- and kernel-phases

Better performance of redundant array over NRM for high contrast binary detection.

Initial assumptions revised

Linear approximation relies on small phase errors

Ongoing extensive simulations suggest kernel-phase on highly redundant aperture is surprisingly robust.

	0	C	D	E	F	G	н	1		к	L	M
30	Hexagonal	ann hex15	78	378	339	258.4 (27.31)	21.1 (9.15)	0.463		47.0 (7.38)	250.9 (30.59)	162.3 (14
31	Hexagonal	ann hex15 w05	48	378	354	152.3 (17.61)	1.0 (0.0)	0.783		26.1 (4.42)	138.0 (13.16)	77.0 (12
32		colav9	9	36	28	38.7 (2.84)	1.0 (0.0)	0.630		5.8(0.0)	10.5 (0.99)	10.7 (2
33	chases)					41.6 (2.92)				48.8 (4.18)	50.6 (4.45)	39.8 (3
34												
35												
36						Unwrapping off, no low freqs		Kernel phases mean STE (dep	rees)			
37	Hexagonal	full hex15	199	378	279	2390 3 (232 92)	2497.2 (829.78)	0.060		1369.9 (146.51)	2606.9 (236.4)	2282.0 (228
38	Hexagonal	ann hex15	78	378	339	1090.9 (101.93)	423.1 (82.32)	0.113		389.7 (41.32)	1128.3 (113.68)	738.2 (70
39	Hexagonal	ann hex15 w05	48	378	354	574.4 (43.15)	106 5 (30.67)	0.214		77.4 (12.26)	582.7 (56.53)	277.3 (22
40		colav9	9	35	28	263.9 (16.33)	81.9 (7.55)	0.131		11.1 (0.0)	49.1 (4.15)	28.6 (4
41	(hases)					258.5 (12.89)				315.2 (17.1)	332.7 (27,84)	256.1 (14
42												
43												
44						Unwrapping off, no low freqs		Kernel phases mean STE (dep	rees)			
45	Hexagonal	full hex15	199	378	279	8771.0 (866.51)	12000.0 (0.0)	0.019		7633.7 (803.44)	9122.4 (867.86)	8373.9 (892
46	Hexagonal	ann hex15	78	378	339	2959.7 (311.48)	3042.9 (833.78)	0.042		2055.7 (279.02)	2907.7 (303.09)	2891.1 (297
47	Hexagonal	ann hex15 w05	48	378	354	1302.6 (96.27)	614.4 (227.31)	0.097		1019.8 (78.13)	1256.8 (106.3)	1281.8 (119
48		colav9	9	35	28	1118.2 (93.88)	1500.8 (137.97)	0.031		353.5 (30.06)	1271.5 (93.69)	451.8 (24
49	(rases)					1117.4 (91.18)				1266.7 (117.26)	1226.7 (108.74)	1103.5 (70
50												
51												
52						Unwrapping off, no low freqs		Kernel phases mean STE (dep	rees)			
53	Hexagonal	full hex15	199	378	279	29032.4 (2887.38)	60000.0 (0.0)	0.007		23625.8 (2416.89)	28447.8 (2872.13)	27539.8 (2827
54	Hexagonal	ann hex15	78	378	339	7815.5 (746.96)	19985.6 (6036.75)	0.017		7074.4 (710.34)	7265.1 (715.55)	7480.7 (700
55	Hexagonal	ann hex15 w05	48	378	354	3181.9 (257.52)	3558.4 (1326.78)	0.041		2038.5 (255.50)	2961.4 (268.7)	3060.9 (251
56		golay9	9	36	28	4549.0 (567.43)	22009.5 (4197.5)	0.008		4036.9 (553.65)	3953.2 (566.54)	4392.3 (642
57	chases)					4489.0 (489.42)				4236.3 (460.92)	4357.9 (501.39)	4525.2 (627
58												
59												
60						Unwrapping off, no low freqs		Kernel phases mean STE (dep	rees)			
61	Hexagonal	full hex15	199	378	279	66290.3 (6418.11)	120000.0 (0.0)	0.003		53702.4 (5750.76)	65668.3 (6231.02)	62210.4 (6730
62	Hexagonal	ann_hex15	78	378	339	16961.7 (1602.45)	94086.3 (24825.23)	0.008		15897.5 (1687.9)	16171.1 (1669.26)	16784.8 (1692
63	Hexagonal	ann hex15 w05	48	378	354	6903.4 (554.81)	17871.1 (6679.32)	0.019		6626.6 (507.53)	6726.1 (545.03)	6850.5 (604
64		golay9	9	36	28	10548.3 (1482.8)	30000.0 (0.0)	0.003		9682.9 (1253.02)	9714.3 (1173.27)	10689.0 (1568
65	(anaste		-			10670 2 (1501 64)				10322 8 (1365 58)	10090 1 (5486.06)	10690.0 (141)

Latyshev, et al, 2015, in prep

Extensions are possible:

Original linearization for small instrumental phase:

 $\Phi_{j} = \Phi_{0j} + \operatorname{Arg}(\Sigma e^{j \Delta \varphi i}) \qquad \qquad \Phi_{j} = \Phi_{0j} + 1/n_{j} \Sigma_{i} \Delta \varphi i$

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alternate linearization scheme: $\Phi_{j}(\lambda) = \Phi_{0j}(\lambda) + \operatorname{Arg}(\Sigma \exp(i2\pi\delta/\lambda))$ $\operatorname{Arg}(\gamma_{j}(\lambda_{1}) \times \gamma^{*}_{j}(\lambda_{2})) = \Delta \Phi_{0}(\lambda_{1},\lambda_{2}) + 1/n \Sigma [2\pi\delta_{j}/\Lambda_{0}]$

The same model holds for differential phase

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It is all about exploiting the properties of A

Martinache, 2010, ApJ, 724, 464 Martinache, 2013, PASP, 125, 422



$K \phi = K \phi o + K \phi \phi$ $K \phi = K \phi o$ (kernel-phase)

$\varphi = A^{-1} \cdot (\varphi - \varphi_{o})$ (eigen-phase)

It is all about exploiting the properties of A

Martinache, 2010, ApJ, 724, 464 Martinache, 2013, PASP, 125, 422